

GCE Examinations  
Advanced Subsidiary / Advanced Level  
**Further Pure Mathematics**  
**Module FP1**

Paper B  
**MARKING GUIDE**

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks should be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for knowing and using a method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.



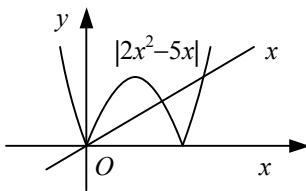
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## FP1 Paper B – Marking Guide

1.



B2

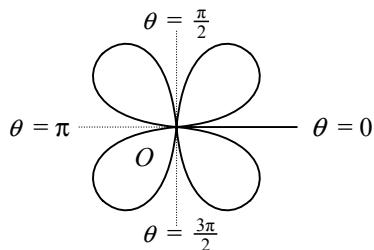
considering  $2x^2 - 5x = x$  gives  $2x(x - 3) = 0 \therefore x = 0, 3$  M1 A1

considering  $-(2x^2 - 5x) = x$  gives  $2x(x - 2) = 0 \therefore x = 0, 2$  A1

using graphs, require  $2 < x < 3$  A1 (6)

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2. (a)



B3

(b)  $\frac{1}{2} \int_0^{\frac{\pi}{2}} a^2 \sin^2 2\theta \, d\theta$  M1

$$= \frac{1}{4} a^2 \int_0^{\frac{\pi}{2}} 1 - \cos 4\theta \, d\theta \quad \text{M1}$$

$$= \frac{1}{4} a^2 [\theta - \frac{1}{4} \sin 4\theta]_0^{\frac{\pi}{2}} \quad \text{A1}$$

giving  $\frac{1}{8} a^2 \pi$  M1 A1 (8)

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3.

(a)  $= \sum_{r=1}^n (r^3 - r^2 + r - 1)$  M1

$$= \frac{1}{4} n^2(n+1)^2 - \frac{1}{6} n(n+1)(2n+1) + \frac{1}{2} n(n+1) - n \quad \text{M1 A1}$$

$$= \frac{1}{12} n[3n(n^2 + 2n + 1) - 2(2n^2 + 3n + 1) + 6(n+1) - 12] \quad \text{M1}$$

$$= \frac{1}{12} n[3n^3 + 2n^2 + 3n - 8] \quad \text{A1}$$

$$= \frac{1}{12} n(n-1)(3n^2 + 5n + 8) \quad \text{A1}$$

(b)  $\sum_{r=5}^{25} (r^2 + 1)(r - 1) = \sum_{r=1}^{25} (r^2 + 1)(r - 1) - \sum_{r=1}^4 (r^2 + 1)(r - 1)$  M1

$$= \frac{1}{12} .25.24.2008 - \frac{1}{12} .4.3.76 \quad \text{M1}$$

$$= 100\ 400 - 76 = 100\ 324 \quad \text{A1}$$

(9)

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4.

(a) int. fac. =  $e^{\int -\cot x \, dx} = e^{-\ln|\sin x|} = \operatorname{cosec} x$  M1 A1

$$\therefore \operatorname{cosec} x \frac{dy}{dx} - y \operatorname{cosec} x \cot x = \operatorname{cosec} x \times 2 \sin x \cos x \quad \text{M1}$$

$$\frac{d}{dx}(y \operatorname{cosec} x) = 2 \cos x \quad \text{A1}$$

$$y \operatorname{cosec} x = \int 2 \cos x \, dx$$

$$y \operatorname{cosec} x = 2 \sin x + c \quad \text{or} \quad y = 2 \sin^2 x + c \sin x \quad \text{M1 A1}$$

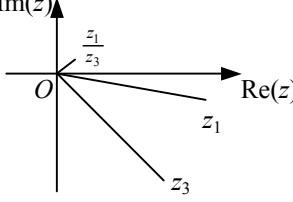
(b)  $x = \frac{\pi}{6}, y = 2 \therefore c = 3$  M1 A1

$$\therefore \text{when } x = \frac{2\pi}{3}, y = \frac{3}{2}(1 + \sqrt{3}) \quad \text{A1}$$

(9)

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5. (a)  $f(1.0) = -0.0986$ ;  $f(1.1) = 0.305$   
 $f$  cont. over interval, change of sign  $\therefore$  root M1 A1
- (b)  $f'(x) = 3x^2 + \frac{2x}{4-x^2}$  M1 A1  
 $x_{n+1} = x_n - \frac{x_n^3 - \ln(4-x_n^2)}{3x_n^2 + \frac{2x_n}{4-x_n^2}}$  M1 A1  
 $x_0 = 1.0, x_1 = 1.026894, x_2 = 1.026221$  A2  
 $f(1.0262205) = -2.02 \times 10^{-6}$ ;  $f(1.0262215) = 1.83 \times 10^{-6}$   
change of sign  $\therefore$  root  $\therefore \alpha = 1.026221$  correct to 6dp M1 A1 (10)
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6. (a)  $|z_1| = \sqrt{(49+1)} = \sqrt{50} = 5\sqrt{2}$  B1  
 $5\sqrt{2} \times |z_3| = 50 \therefore |z_3| = 5\sqrt{2}$  M1 A1
- (b)  $\arg z_2 = \tan^{-1} \sqrt{3} = \frac{\pi}{3}$  B1  
 $\frac{\pi}{3} - \arg z_3 = \frac{7\pi}{12} \therefore \arg z_3 = -\frac{\pi}{4}$  M1 A1
- (c)  $z_3 = 5\sqrt{2}[\cos(-\frac{\pi}{4}) + i \sin(-\frac{\pi}{4})] = 5 - 5i \therefore a = 5, b = -5$  M1 A1
- (d)  $\frac{z_1}{z_3} = \frac{7-i}{5-5i} \times \frac{5+5i}{5+5i} = \frac{40+30i}{50} = \frac{1}{5}(4+3i)$  M2 A1
- (e)  B2
- (f) e.g.  $|z_1| = |z_3| \therefore |\frac{z_1}{z_3}| = |\frac{z_3}{z_1}|$ ;  $\arg \frac{z_1}{z_3} = \arg z_1 - \arg z_3 = -\arg \frac{z_3}{z_1}$   
same modulus, -ve of argument  $\therefore$  conjugate M1 A1 (15)
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7. (a)  $\frac{dx}{dt} = e^t; \frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} = e^{-t} \frac{dy}{dt}$  M1 A1  
 $\frac{d^2y}{dx^2} = \frac{dt}{dx} \left( e^{-t} \frac{d^2y}{dt^2} - e^{-t} \frac{dy}{dt} \right) = e^{-2t} \left( \frac{d^2y}{dt^2} - \frac{dy}{dt} \right)$  M1 A2
- (b)  $e^{2t} \cdot e^{-2t} \left( \frac{d^2y}{dt^2} - \frac{dy}{dt} \right) - e^t \cdot e^{-t} \cdot \frac{dy}{dt} - 3y = 6 \cdot e^{2t}$  M1 A1  
giving  $\frac{d^2y}{dt^2} - 2 \frac{dy}{dt} - 3y = 6e^{2t}$  A1
- (c) aux. eqn.  $m^2 - 2m - 3 = 0$  M1  
 $(m+1)(m-3) = 0; m = -1, 3$  C.F.  $y = Ae^{-t} + Be^{3t}$  A1  
for P.I. try  $y = Ce^{2t} \therefore \frac{dy}{dt} = 2Ce^{2t}, \frac{d^2y}{dt^2} = 4Ce^{2t}$  M1  
so  $4Ce^{2t} - 4Ce^{2t} - 3Ce^{2t} = 6e^{2t} \therefore C = -2$  M1 A1  
gen. soln.  $y = Ae^{-t} + Be^{3t} - 2e^{2t} = \frac{A}{x} + Bx^3 - 2x^2$  A1  
 $\frac{dy}{dx} = -\frac{A}{x^2} + 3Bx^2 - 4x$  M1  
 $x = 1, y = 3 \therefore 3 = A + B - 2$   
 $x = 1, \frac{dy}{dx} = -5 \therefore -5 = -A + 3B - 4$  giving  $A = 4, B = 1$  M1 A1  
 $\therefore y = x^3 - 2x^2 + \frac{4}{x}$  A1 (18)
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Total (75)

## **Performance Record – FP1 Paper B**